EXAMPLE 3A

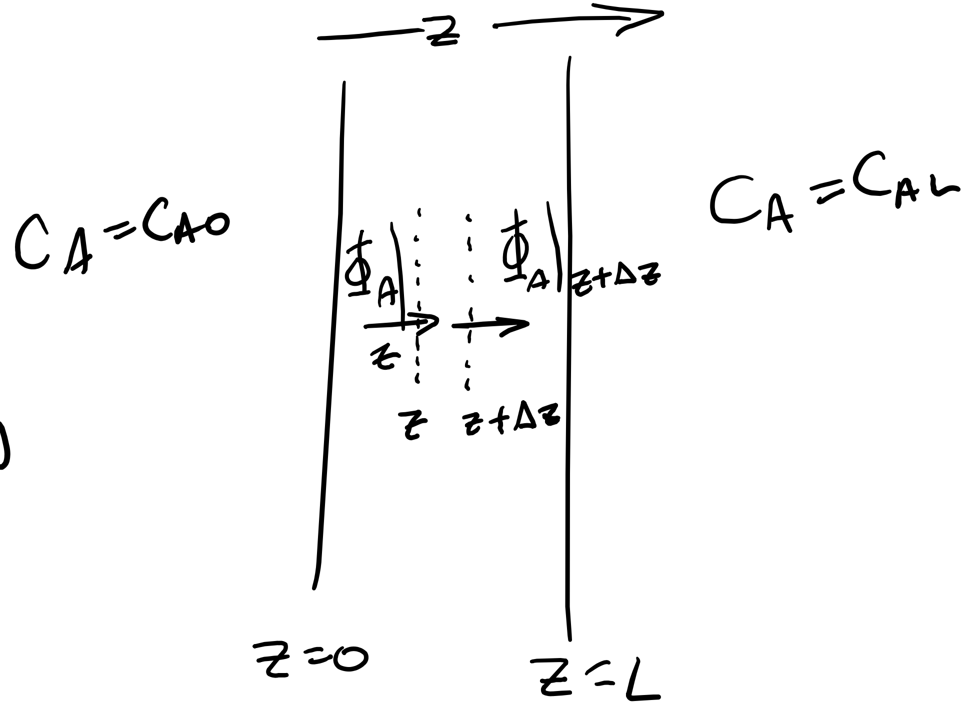
STEADY-STATE DIFFUSION OF "A" ACROSS A THIN FILM OF "B".

"FILM" = SOLID "B" ACTS MERELY AS A BOUNDARY BETWEEN TWO SOLUTIONS. THERE IS NO INTERACTION BETWEEN "A" & "B".

STEP 1

Let's say that $C_{A0} > C_{AL}$

DIFFUSION IS IN +z DIRECTION



STEP 2

MATERIAL BALANCE (EX. 1A)

$$\frac{d\Phi_A}{dz} = 0 \quad \text{OR} \quad \Phi_A = C_1$$

STEP 3

FLUX EQN

DIFFERENT IN THE 3 COORDINATE SYSTEMS.

$$\vec{\Phi}_A = -cD_{AB}\boxed{\vec{\nabla} x_A} + \underbrace{x_A \sum \vec{\Phi}_A}_{\cong 0}$$

- DIFFUSION IS IN Z-DIRECTION ONLY
- SOLID "B" IS STAGNANT ($\Phi_B = 0$)
- ASSUME "A" IS VERY DILUTE $x_A \sim 0$

  c IS CONSTANT

$$\Phi_A = -cD_{AB}\frac{dx_A}{dz} = -D_{AB}\frac{dc_A}{dz}$$

$$-D_{AB}\frac{dc_A}{dz} = C_1$$

INTEGRATE AGAIN...

$$-D_{AB}c_A = C_1 z + C_2$$

STEP 4

APPLICATION OF BOUNDARY CONDITIONS

1) $c_A = c_{A0}$ @ $z=0$ $\qquad -D_{AB}c_{A0} = C_2$

2) $c_A = c_{AL}$ @ $z=L$ $\qquad -D_{AB}c_{AL} = C_1 L + C_2$

$$-D_{AB}c_{AL} = C_1 L - D_{AB}c_{A0}$$

$$C_1 = \frac{D_{AB}(C_{A0} - C_{AL})}{L}$$

$$-D_{AB} C_A = D_{AB}(C_{A0} - C_{AL})\frac{z}{L} - D_{AB} C_{A0}$$

$$\boxed{C_A = C_{A0} - \frac{z}{L}(C_{A0} - C_{AL})}$$

CONC PROFILE

$$\boxed{\Phi_A = C_1 = \frac{D_{AB}(C_{A0} - C_{AL})}{L}}$$

FLUX

EXAMPLE 3B

STEADY-STATE DIFFUSION OF "A" ACROSS MEMBRANE OF "B".

"MEMBRANE" = SOLUTE "A" INTERACTS WITH "B" AND "A" IS IN EQUILIBRIUM BETWEEN SOLVENT & SOLID PHASE. THE EQUILIBRIUM IS DEFINED BY A PARTITION COEFFICIENT.

$$H = \frac{\text{EQUIL. CONC. OF SOLUTE "A" IN SOLID PHASE}}{\text{EQUIL. CONC. OF SOLUTE "A" IN FLUID PHASE}}$$

NOTE SIMILARITY BETWEEN PARTITION COEFFICIENT & HENRY'S LAW COEFFICIENT:

STROLL DOWN MEMORY LANE.

HENRY'S LAW CONSTANT = $H_i^* = \frac{Y_i P}{X_i}$

$$\frac{Y_i}{X_i} = \frac{\text{EQ MOLE FRACTION OF SOLUTE IN GAS PHASE}}{\text{EQ MOLE FRACTION OF SOLUTE IN LIQ PHASE}}$$

IF $H > 1$ SOLUTE IS MORE SOLUBLE IN MEMBRANE THAN IN FLUID

IF $H < 1$ SOLUTE IS MORE SOLUBLE IN FLUID THAN IN MEMBRANE.

THIS PROBLEM IS IDENTICAL TO EXAMPLE 3A

EXCEPT FOR BOUNDARY CONDITIONS:

@ $Z=0$ $\quad H = \dfrac{C_{A0}^{MEM}}{C_{A0}}$ $\quad C_{A0}^{MEM} = HC_{A0}$

@ $Z=L$ $\quad H = \dfrac{C_{AL}^{MEM}}{C_{AL}}$ $\quad C_{AL}^{MEM} = HC_{AL}$

STEP 4

1) $C_A = HC_{A0}$ @ $Z=0$

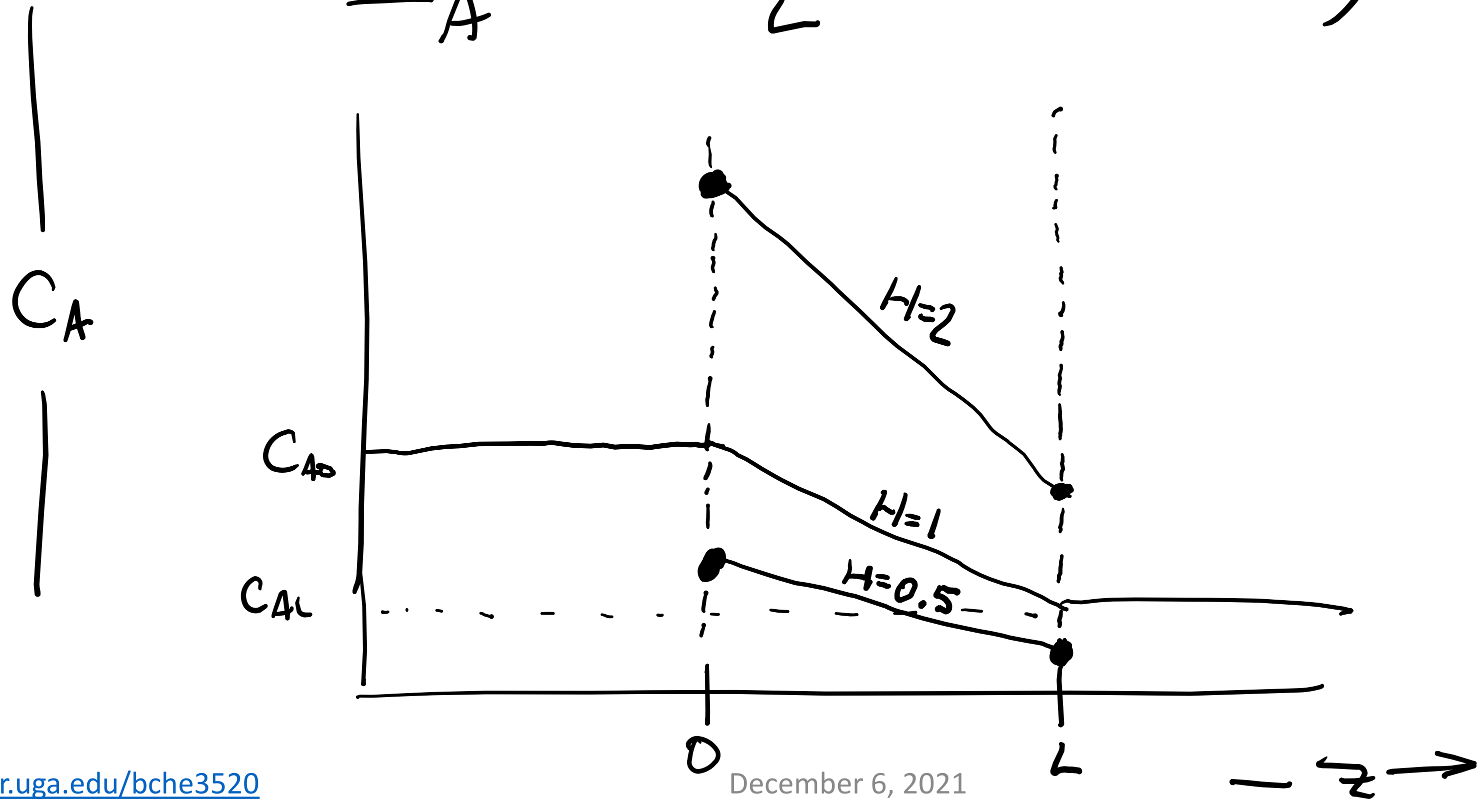
2) $C_A = HC_{AL}$ @ $Z=L$

CONC. PROFILE

$$C_A = H C_{A0} - \frac{z}{L} H \left(C_{A0} - C_{AL} \right)$$

FLUX

$$\Phi_A = \frac{D_{AB} H}{L} \left(C_{A0} - C_{AL} \right)$$



IF COMPONENT "A" IS A GAS, IT IS MORE CONVENIENT TO RELATE FLUX WITH PRESSURE OF "A"

$$P_A = C_A R T \qquad \text{IDEAL GAS LAW}$$

$$C_A = \frac{P_A}{RT}$$

$$J_A = \frac{D_{AB} H}{L} \frac{(P_{A0} - P_{AL})}{RT}$$

$\rightarrow$ PERMEABILITY $= P_{AB}$ $[m^3 \text{ GAS @ STP} / s \cdot m \cdot atm]$

0°C, 1atm

$$\Phi_A = \frac{P_{AB}}{L} \frac{(P_{A0} - P_{AL})}{RT}$$

PRESSURES

PERMEABILITY

RT @ STP = 22.4 L/mol

22.4 m^3/Kmol

COMMENTS

- EQUATIONS WORK WELL FOR CELLULOSE, RUBBER, POLYMERS
- EQUATIONS POOR FOR LEATHER, FIBROUS BOARDS
- EQUATIONS INVALID FOR GLASS & METALS (SMALL PORE SIZE)